APPENDIX B: PROPERTIES OF COURNOT MODELS

A general property of Cournot models is that firms charge a mark-up over marginal cost in equilibrium, where each firm's mark-up is related to the price elasticity of demand in the market and its own market share. To see this, consider the profit maximization problem faced by a given firm, known as firm i. Let p(Q) represent the inverse demand function, which describes the price needed to clear the market when total market output equals Q. Since consumers will not purchase additional output unless the price of the good declines, it holds that the partial differential $p'(Q) \equiv dp/dq < 0$. Next, let q_i represent the quantity produced by firm i, and $c_i(q_i)$ represent the production costs for firm i based on an output level, q_i. The profit maximization problem facing firm i is as follows:

$$max_{qi}\ p^i(q_i,Q_i) = p(q_i + Q_i)\ q_i - c_i(q_i),$$

where Q_i represents the output of all producers other than firm *i*. Assuming that p and c_i are differentiable functions, then the first-order condition for an optimal solution is:

$$p(q_i + Q_i) + p¢\ (q_i + Q_i)\ q_i - c_i¢(q_i) = 0,$$

where ¢ denotes the first derivative of the corresponding function.

In this equation, the first two terms on the left-hand side of the first equality, $p(q_i + Q_i) + p¢\ (q_i + Q_i)\ q_i$, represent the marginal revenue from an additional unit of output, while the third term, $c_i¢(q_i)$, represents the marginal cost of that output. Thus, the first-order condition for profit-maximization requires that marginal revenue equal marginal cost. The marginal revenue term, $p(q_i + Q_i) + p¢\ (q_i + Q_i)\ q_i$, equals the price of each unit of output, $p(q_i + Q_i)$, plus the decline in market price needed to sell an additional nit of output, multiplied by the amount of output produced by firm *i*, $p¢\ (q_i + Q_i)\ q_i$. In a Cournot model, firms choose their output while taking rival output choices as given. Consequently, in maximizing its profits, a given firm assumes that an increase in its own output increases market output by an equivalent amount.

Referring back to the equation, and letting p denote price, and dropping the arguments of each function for notational simplicity, this equation can be rewritten as

$$p - c_i' = - p' q_i > 0.$$

This demonstrates that profit-maximizing behavior in Cournot model requires that price exceeds marginal cost, where the price-marginal cost markup equals the absolute value of the firm's output multiplied by the slope of the inverse demand function. This last equation can be rewritten in order to better develop our result on the effect of price increases. Start with the transformation:

$$p' \ q_i = (p/p)(p' \ q_i)(Q/Q) = p(p' \ Q/p)(q_i/Q) = -p(1/e) \ s_i,$$

where e represents the price elasticity of market demand, and s_i equals firm i's market share. Substituting this result into the previous equation yields the result:

$$p - c_i' = p(s_i /e)$$

or

$$(p - c_i')/p = s_i /e.$$

This demonstrates that in a Cournot-Nash equilibrium, the equilibrium price-marginal cost margin for each firm equals the inverse of the price elasticity of market demand times that firm's equilibrium market share. In other words, each firm "perceives" that it faces a price elasticity of demand equal to the market's price elasticity of demand divided by that firm's own market share. To maximize its profits each firm adjusts its output until its price-marginal cost margin equals the inverse of its "perceived" price elasticity of demand.

Now consider the situation when the marginal cost changes for all firms. Let a_i be the percentage amount that the marginal cost changes for firm i. So, $a_i c_i'$ is the marginal cost for firm i after the marginal cost increases. Let $\underline{p}$ be the equilibrium price and $\underline{s}_i$ firm i's market share after the marginal cost changes. Solving the last equation for s_i, yields

$$e\ (p - c_i¢)/p = s_i \text{ and } e\ (\underline{p} - a_i c_i¢)/\underline{p} = \underline{s}_i \text{ for each firm } i.$$[1]

Since $\Sigma\ s_i = 1$ and $\Sigma\ \underline{s}_i = 1$, one can write

$$S\ e\ (p - c_i¢)/p = S\ e\ (\underline{p} - a_i c_i¢)/\underline{p}.$$

Rearranging terms and dividing by *e* yields

$$(p - S\ c_i¢)/p = (\underline{p} - S a_i c_i¢)/\underline{p}$$

and then

$$\underline{p}(p - S\ c_i¢) = p(\underline{p} - S a_i c_i¢).$$

Finally, subtracting $\underline{p}p$ and rearranging again yields

$$\underline{p}/\ p = S a_i c_i¢/S\ c_i¢.$$

So the result is that the price increase due to an increase in costs at the margin for each firm is given by the weighted sum of the new marginal costs divided by the sum of the old marginal costs. If $a = a_i$ is for all firms i, then the increase in costs is a, that is $\underline{p} = ap$. Therefore, when all costs change at the margin, there is a proportionate increase in price.

If some of the firms costs at the margin do not change, then that means some of the $a_i = 1$. The only difference in the result is that the increase could be less than one-for-one. As a result, even if the marginal bidder in the PX does not experience an increase in its gas prices, any seller into the PX that has gas as its own fuel and experiences a gas cost increase causes an increase in the PX price.

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[1] Here we assume that the elasticity of demand is the same at each equilibrium. This can happen in a number of ways. One way is to use a constant elasticity of demand function. This assumption is widely accepted for Cournot models for electric power. *See* Borenstein-Bushnell, etc. A technical adjustment so that the demand function is not infinite at zero must be made so that the model is actually solvable, but that does not materially affect the result here.